Problems on complex numbers

1.

A. define the complex numbers geometrically.

B. Show that if $x + ib = c + id$ iff $x = c \; and \; b = d$.

2. Prove the following properties:

A. $\overline{z^n} = \bar{z}^n$

B. $\bar{\bar{z}} = z$

*try to prove them geometrically.

3. Prove the following property:

A. Based on your geometric definition of complex number every number can be represented as:

$$z = R(cos\theta + isin\theta)$$

B. If $z_1, z_2 \in \mathbb{C}$ and $z_1 = r_1(cos\theta_1 + isin\theta_1)$, $z_2 = r_1(cos\theta_2 + isin\theta_2)$ then $z_1 z_2 = r_1 r_2\big(\cos(\theta_1 + \theta_2) + isin(\theta_1 + \theta_2)\big)$.

3. Find the group of numbers in the *Cartesian plane*[1] of the graph $f(x) = 1/x$ rotated by $45°$.

4. Prove that if $p(z) = 0$ where $p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$ and $a_i \in \mathbb{R}$,then $p(\bar{z}) = 0$.

A harder version: try to prove (Q4) geometrically.

[1] *Cartesian plane* -The regular x, y plane.